6.1 - Capacitors

Core Info and Definitions

A capacitor is a circuit component that stores charge in a circuit by separating equal and opposite charges onto two electrical conductors (plates) with an insulator in between them.

Capacitance, C, defines the quantity of charge Q which can be stored per unit potential difference across the plates. Measured in farads

Capacitance

$C = QV = \frac{\epsilon_0 \epsilon_r A}{d}$

... where epsilon 0 is the permittivity of free space, epsilon r is relative permittivity of the material, A is the surface area of a plate, and d is the separation between the plates.

Capacitance of an Isolated Conducting Sphere:

$$C = \frac{Q}{V}$$

$$V = \frac{Q}{4\pi\epsilon_0 R}$$

$$Thus,$$

$$C = Q \times \frac{4\pi\epsilon_0 R}{Q}$$

$$C = 4\pi\epsilon_0 R$$

... where R is the radius of the sphere.

Total Circuit Capacitance

In series, capacitance sum is determined in a similar way to how resistance is determined with resistors in parallel:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$

In parallel, it's the opposite - the capacitances are just added to each other:

$$C = C_1 + C_2 + C_3 + \cdots + C_n$$

Capacitor Energy

Capacitor energy can be given as:

$$\begin{aligned} W &= \int_0^Q V\, dQ \\ &= \int_0^Q \frac{Q}{C}\, dQ \\ &= \frac{Q^2}{2C} \\ &= \underline{\mathbf{\frac{1}{2}QV}} \end{aligned}$$

... and you can use Q=CV to substitute values in to determine energy with charge.

(You do not need to know the integral derivation. It is only for explanation.)

The work done in moving a charge Q from one plate to another through a **constant potential difference** V is:

$$\begin{aligned} \Delta W &= \int_0^{\Delta Q} V\, dQ \\ &= \underline{\mathbf{\Delta QV}} \end{aligned}$$

The reason the two energy equations are different is because **V is constant in the latter**, but **not the former**. Capacitance is a constant for each capacitor, however.

A capacitor can leak charge . This is because the insulator between the plates is not perfect, so there is a tiny current that passes through them. This charge leakage is more clearly observable when disconnecting a capacitor from the source of emf.

Charging and Discharging

Where x_0 is the initial value of the variable, C is capacitance, R is resistance and t is elapsed time:

	Current	**Charge**	**PD**

Charging	$I = I_0 e^{\frac{-t}{CR}}$	$Q = Q_0(1 - e^{\frac{-t}{CR}})$	$V = V_0(1 - e^{\frac{-t}{CR}})$
Discharging	$I = I_0 e^{\frac{-t}{CR}}$	$Q = Q_0 e^{\frac{-t}{CR}}$	$V = V_0 e^{\frac{-t}{CR}}$

(... yes, current has the same equation for charging and discharging.)

The time taken for the charge of the capacitor to fall to 1/e (~37%) of its original charge is known as the time constant.

$$\tau = CR$$

As more charge is added to a capacitor, it gets harder to add more charge to it. This concept is analogous to pumping a car tyre - it is initially easy to add air to it, but the increase of internal pressure makes adding more harder and harder.

Graphical Methods

Take V in discharging for an example. You can apply the natural logarithm "ln" to both sides to separate variables and obtain a straight line in a graph:

$$V = V_0 e^{\frac{-t}{CR}} \rightarrow ln(V) = ln(V_0) + \frac{-t}{CR}$$

In here, the y-intercept is "ln(V_0)" and the gradient is "-1/CR". You can use the gradient to find the capacitance of the circuit, and you can use the y-intercept to find the initial pd of the capacitor by raising e to the power of "ln(V_0)".

Spreadsheet Modelling

You can model the discharge of a capacitor using a spreadsheet method without using experimental data. This is known as iterative modelling. You can do this with the following equation:

$$\Delta Q = \frac{Q}{CR} \times \Delta t$$

This gives the decrease of charge, so the output of this should be subtracted from the last value of charge. The table looks like this[1]:

t/s	ΔQ / μC	Q / μC
0.0	20	1000
0.1	19.6	980.0
0.2	19.2	960.4
0.3	18.8	941.2
0.4	18.4	922.4

... with a time increment of +0.1.

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